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Roll No. | | | | | | | | | | |

S. No. of Question Paper : **2827**

Unique Paper Code : **32371402** **GC-4**

Name of the Paper : **Linear Models**

Name of the Course : **B.Sc. (H) Statistics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all,

selecting *three* questions from each Section.

Section I

1. State and prove Cochran's theorem. 12½

2. Derive the analysis of variance for two-way classified data with one observation per cell under fixed effect model. 12½

P.T.O.

3. (*a*) Consider $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$, $i = 1, 2, j = 1, 2, 3$. Write the normal equations for estimating μ and α_i. Are $\mu + \alpha_1$, $\alpha_1 + \alpha_2$, $\alpha_1 - \alpha_2$, $\mu + \alpha_1 + \alpha_2$ estimable and why ?

(*b*) Describe the general linear model and discuss briefly the different studies derived from it. 6½,6

4. (*a*) Consider the model $E(Y_1) = 2\beta_1 + \beta_2$, $E(Y_2) = \beta_1 - \beta_2$, $E(Y_3) = \beta_1 - \beta_3$ with usual assumptions. Obtain the BLUE of $\beta_1 + 2\beta_2$ and its variance.

(*b*) For a given model $\underset{\sim}{Y}_{n\times 1} = X_{n\times p}\underset{\sim}{\beta}_{p\times 1} + \underset{\sim}{\in}_{n\times 1}$ with $E(\underset{\sim}{\in}) = \underset{\sim}{0}$, $V(\underset{\sim}{\in}) = \sigma^2 I$ and $\rho(X) = p < n$. Obtain an unbiased estimator of σ^2. 7½,5

Section II

5. (*a*) For a Simple linear regression model $Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$, find the 100 $(1 - \alpha)$% confidence interval for the mean response at a particular value of the regressor variable X.

(*b*) Show that, for any linear model :

$$\sum_{i=1}^{n} V(\hat{Y}_i)/n = trace\,\frac{\{X(X'X)^{-1}X'\}\sigma^2}{n} = p\sigma^2/n$$

where p is the number of unknown parameters. 6,6½

6. (*a*) Discuss the problem of testing for lack of fit in simple linear regression model.

(*b*) What do you mean by bias in regression estimates ? Suppose we postulate the model $E(Y) = \beta_0 + \beta_1 X$ but the model $E(Y) = \beta_0 + \beta_1 X + \beta_{11} X^2$ is actually the true response function, unknown to us. If we use observations of Y at $X = -1, 0, 1$ to estimate β_0 and β_1 in the postulated model, what biases will be introduced in them ? 6,6½

7. (*a*) Which specific regressors seem important in multiple regression ? How will you address this question ? Discuss.

P.T.O.

(*b*) We fit a straight line model to a set of data using the formulas $b = (X'X)^{-1}X'Y$, $\hat{Y} = Xb$ with the usual definitions. We define $H = X(X'X)^{-1}X'$. Show that :

SS(due to regression) = $Y'HY = \hat{Y}'\hat{Y} = \hat{Y}'H^3\hat{Y}$. 6,6½

8. Write notes on any *two* of the following : 6,6½

(*a*) Role of orthogonal polynomials in fitting polynomial models in one variable

(*b*) Stepwise regression method

(*c*) No intercept regression model.